Maple Tutorial 1: Variables, Expressions and Functions

Notation in this document: Maple commands are in **red**, returned output from Maple is in **blue**.

Always begin a session with the *restart* command to clear previous variables. The colon suppresses output. Here's the command:

restart **:**

1. Variables

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Maple uses **expressions** and a **functions** . As a in Maple is that an expression has an explicit dependence on particular variables, i.e. the expression x^2 is not the same as y^2, while a function specifies a mapping of how an input domain is mapped onto a output range, i.e. the function $f(x) = x^2$ is exactly the same as the function $f(y) = y^2$. In the case of a function, the variable x in $f(x)$ is just a temporary placeholder that is used to define the mapping.

1. Expressions

An expression is a particular combination of variables, such as x + y. Maple lets you assign expressions to a variable like this

$z := x + y$

$$z := x + y \tag{1}$$

You can use expressions to build more complex expressions:

$w := z^3 - z^2$

$$w := (x + y)^3 - (x + y)^2 \tag{2}$$

The subs() command allows you to plug in numeric values into an expression. In the following example we substitute x=1, y=2 and evaluate $z = x + y$

subs **(x = 1, y = 2, z)**

$$3 \tag{3}$$

The next example evaluates w when $z = 3$

subs **(z = 3, w)**

$$18 \tag{4}$$

You can use the subs() command to substitute variables (or even expressions) for other variables as well:

$$a := subs(x = \cos(x), y = \sin(x), w)$$

$$a := (\cos(x) + \sin(x))^3 - (\cos(x) + \sin(x))^2 \quad \textbf{(5)}$$

The evalf() function forces Maple to evaluate the expression as a floating point number:

$$evalf(subs(x = \mathrm{Pi}, a))$$

$$-2.000000000 \quad \textbf{(6)}$$

We can expand polynomial expressions

$$expand((x + y)^4)$$

$$x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 \quad \textbf{(7)}$$

and factor them:

$$factor(4 \cdot x^3 - 3 \cdot x \cdot y^2 + y^3)$$

$$(x + y)(-y + 2x)^2 \quad \textbf{(8)}$$

The simplify() command attempts to reduce an expression to its simplest form:

$$simplify\left(\frac{x^2 - x \cdot y - 2 \cdot y^2}{x - 2y}\right)$$

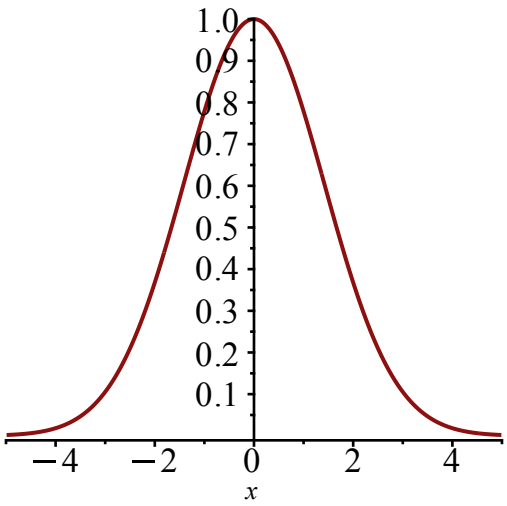
$$x + y \quad \textbf{(9)}$$

Let's see how to plot an expression. First define the expression y in terms of x and then plot it on the interval $(-5, 5)$. Notice that we must include the "x=" when we specify the plot domain.

$$z := \exp\left(-\frac{x^2}{4}\right)$$

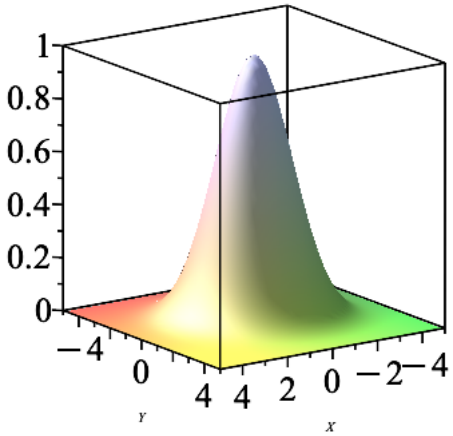
$$z := e^{-\frac{x^2}{4}} \quad \textbf{(10)}$$

$$plot(z, x = -5..5)$$



Here's how to make a surface plot using an expression that depends on x and y. Notice that we use capital X and Y rather than lower case x and y because we have previously defined x and y in other calculations. Using lower case x and y in the following example would give an error. Alternately, we could have just executed a restart command to erase the x and y definitions. Notice that in the following example we just include our function directly in the plot3d command.

```
plot3d(exp(−(X^2 + Y^2)/4), X=−5..5, Y=−5..5)
```



Let's clear the memory to prepare for the next section

```
restart:
```

2. Functions

Functions may be thought of as mappings from an input value to an output value. Consider, for example, the function $f(x) = x^2$. To create a function we first specify the independent variable (x in this example) and then include the mapping symbol (a hyphen "-" followed by a greater than sign ">"):

$$f := x \to x^2$$

$$f := x \mapsto x^2 \tag{11}$$

We evaluate the function like this

$$f(3)$$

$$9 \tag{12}$$

Functions can contain parameters

$$g := x \to \exp(-a \cdot x)$$

$$g := x \mapsto e^{-a \cdot x} \tag{13}$$

$$g(3)$$

$$e^{-3\,a} \tag{14}$$

If you want to specify a numeric value for a you can define it before evaluating the function

$$a := 2$$

$$a := 2 \tag{15}$$

Now, a will have the value 2 when we call the function g():

$$g(3)$$

$$e^{-6} \tag{16}$$

If you want a floating point number use evalf():

$$evalf(g(3))$$

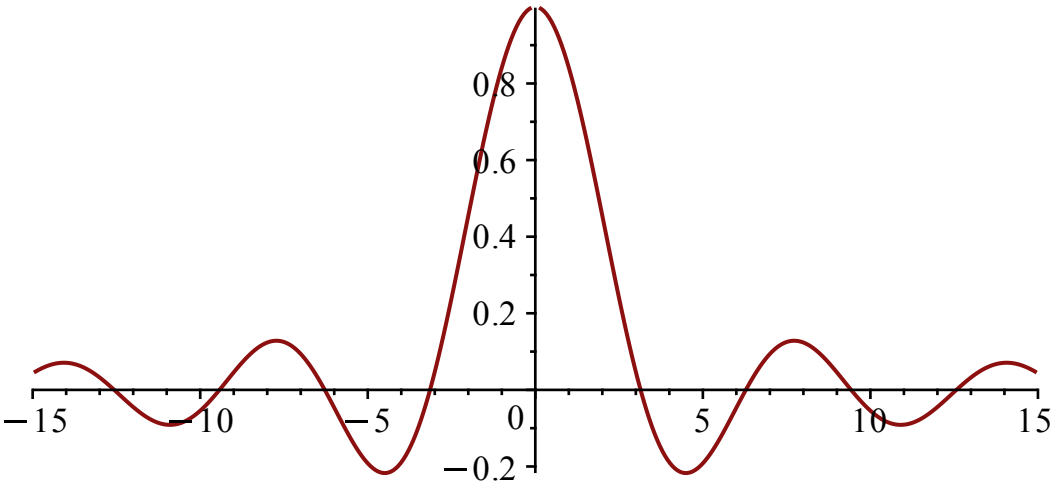
$$0.002478752177 \tag{17}$$

We can plot a function by first defining it, then plotting it like this:

$$g := x \to \frac{\sin(x)}{x}$$

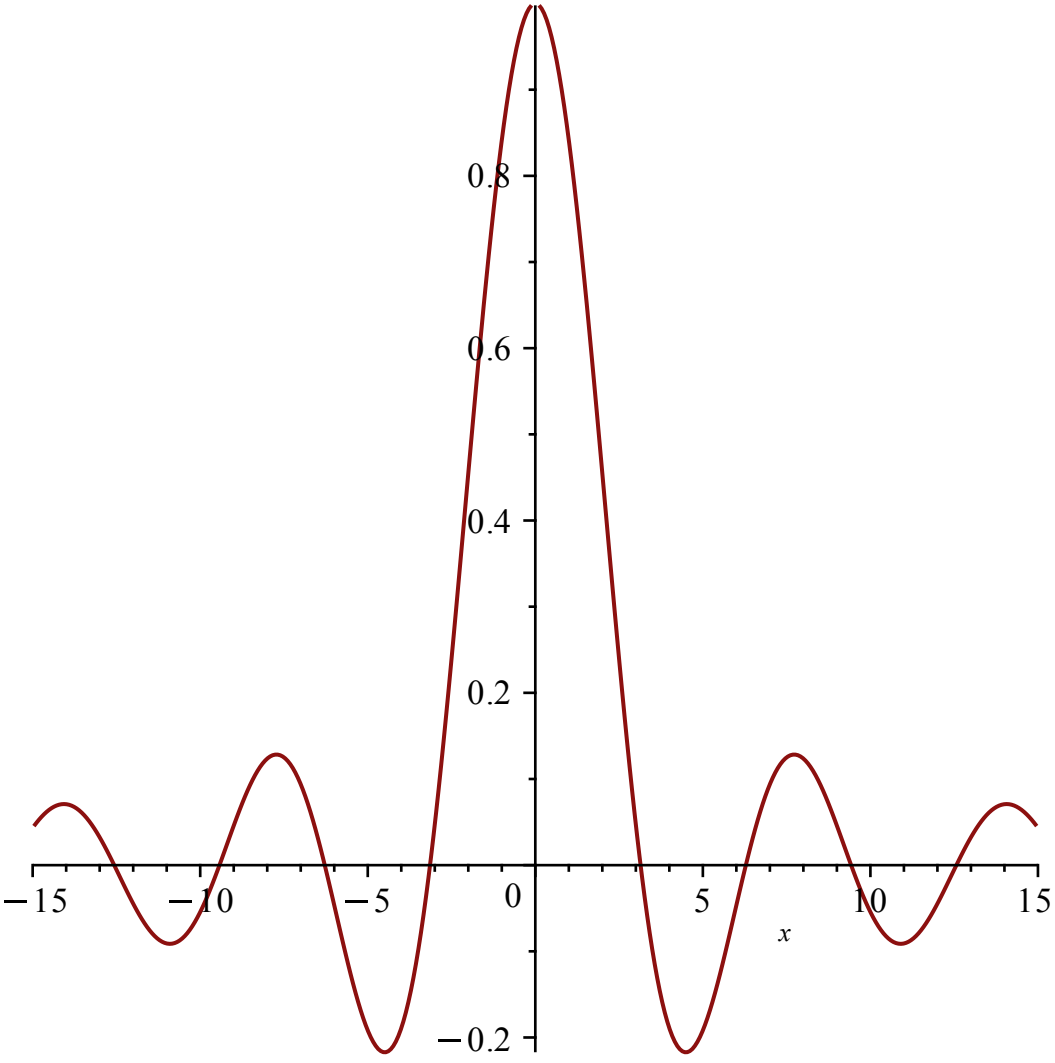
$$g := x \mapsto \frac{\sin(x)}{x} \tag{18}$$

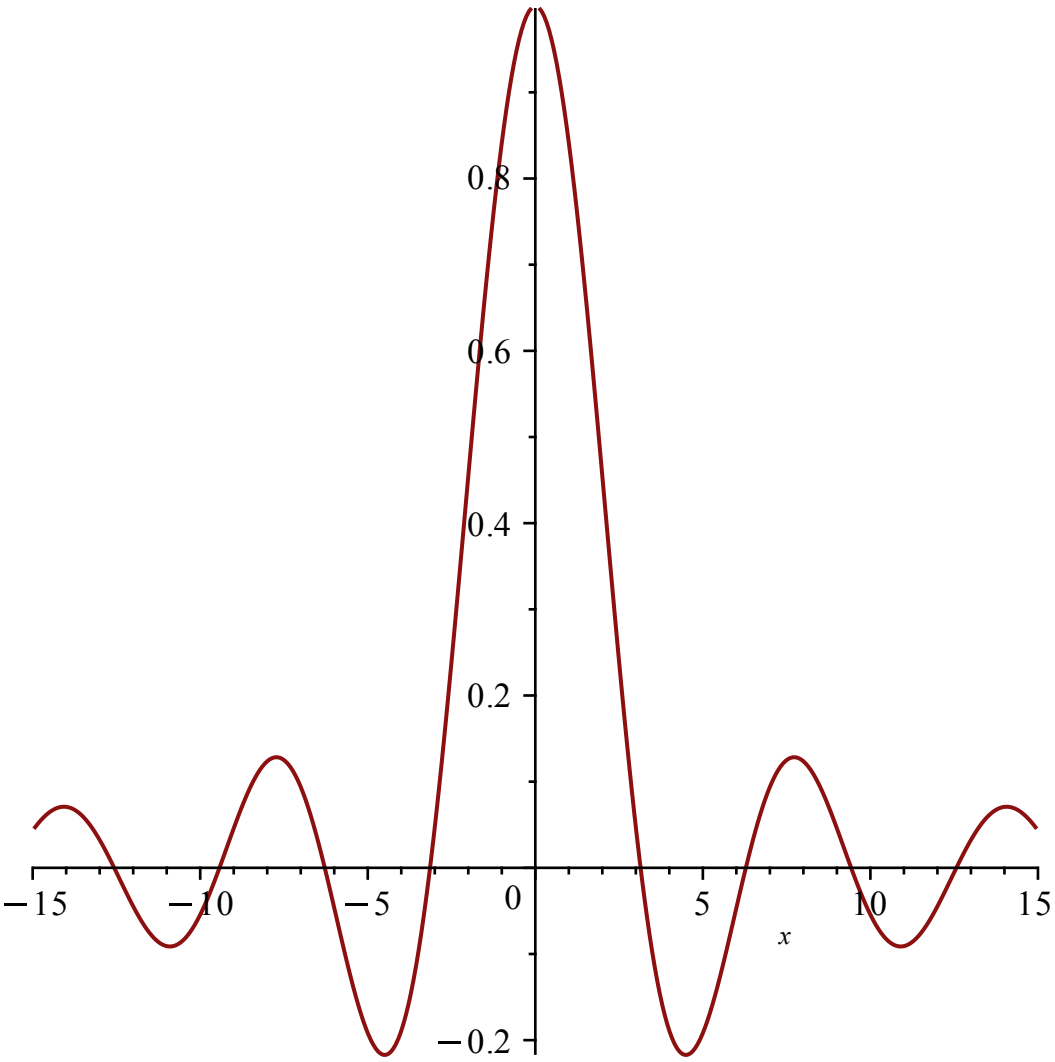
$$plot(g, -15..15)$$



Or if you like to show the x dependence explicity you can try the following (but remove the colon at the end of the statement so it will display on the screen). Technically you are plotting an expression rather than a function here (see below for more info on this).

$plot(g(x), x = -15..15)$:





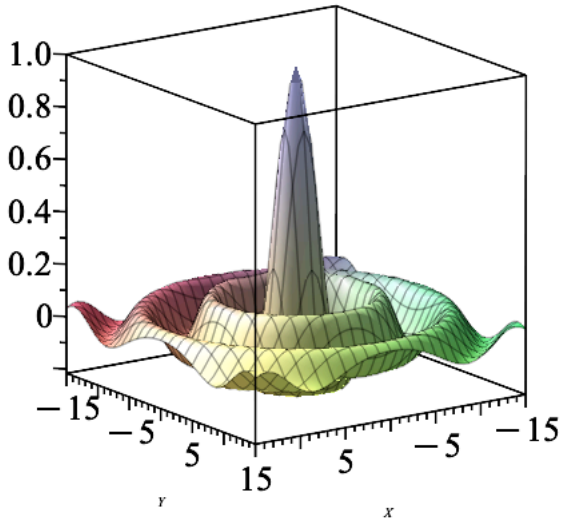
Functions of more than one variable are also possible. Here's a function of two variables:

$$z := (x, y) \to \frac{\sin(\mathrm{sqrt}(x^2 + y^2))}{\mathrm{sqrt}(x^2 + y^2)}$$

$$z := (x, y) \mapsto \frac{\sin\left(\sqrt{y^2 + x^2}\right)}{\sqrt{y^2 + x^2}} \tag{19}$$

And here's an example of how to graph a surface plot using the 2D function defined above

$$plot3d(z(X, Y), X = -15..15, Y = -15..15)$$



3. Concluding Remarks about Expressions and Functions

Suppose you define the following function

$fcn := s \to 4 \cdot s^2$

$$fcn := s \mapsto 4 \cdot s^2 \tag{20}$$

and the following expression:

$expr := 4 \cdot s^2$

$$expr := 4\, s^2 \tag{21}$$

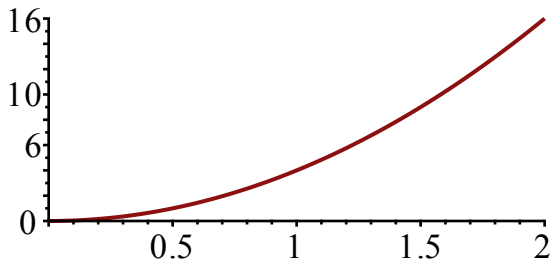
In Maple, when you evaluate the function like this $fcn(t)$ the result is an expression (its no longer a function):

$fcn(t)$

$$4\, t^2 \tag{22}$$

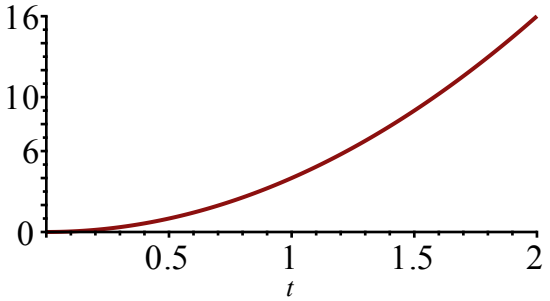
This difference can be confusing at times because operations on functions and expressions have their own unique syntax. Consider the plot command, for example. When you plot a function, you never specify the name of the independent variable:

$plot(fcn, 0..2)$



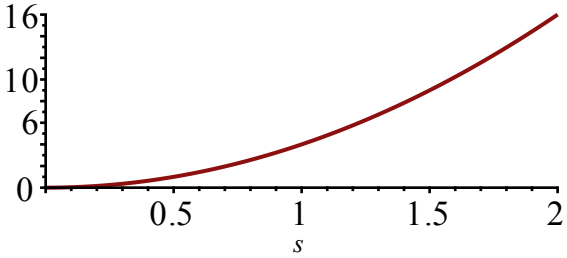
If you explicitly evaluate the function at time t like this $fcn(t)$, then you are technically plotting an expression and not a function, so you must include the name of the independent variable t in the domain like this:

$plot(fcn(t), t = 0..2)$



If you plot the expression *expr*, the syntax looks like

$plot(expr, s = 0..2)$



But the following two combinations will give errors, because you would be using the expression syntax for a function and vise versa, which is not allowed.

$plot(expr, 0..2)$

```
Error, (in plot) procedure expected, as range contains no plotting variable
```

$plot(fcn, t = 0..2)$

```
Error, (in plot) expected a range but received t = 0 .. 2
```

The differences between expressions and functions are even more important when solving differential equations (see next section.)