

## Maple Tutorial 3: solving equations

Notation in this document: Maple commands are in **red**, returned output from Maple is in **blue**.

Always begin a session with the *restart* command to clear previous variables. The colon suppresses output. Here's the command:

**restart :**

### Solve for a variable

Let's say we have an expression for the vertical position of a falling object (released from rest) as a function of time:

$$y = \frac{1}{2} g t^2$$

Suppose we want to solve for time as a function of the distance fallen. The simplest way is to use the **solve** command by writing the equation we want to solve (with just an equals sign " $=$ ", not an assignment operator " $:=$ "). The second argument is the variable we want to solve for, *t* in this example. We will store the solution in a variable called *t<sub>fall</sub>*

$$t_{\text{fall}} := \text{solve}\left(y = \frac{1}{2} g t^2, t\right)$$
$$t_{\text{fall}} := \frac{\sqrt{2} \sqrt{g y}}{g}, -\frac{\sqrt{2} \sqrt{g y}}{g} \quad (1)$$

We see that we have two solutions corresponding the two roots of the square root operator: the first is positive, the second is negative. Let's say we want to pick the first root (the positive one). These solutions are stored as elements in an array, so we can select out the first root like this:

$$t_{\text{fall}} := t_{\text{fall}}[1]$$
$$t_{\text{fall}} := \frac{\sqrt{2} \sqrt{g y}}{g} \quad (2)$$

which leaves us with the positive root.

### Simplifying an equation using constraints

Maple always tries to represent solutions in the most general way possible. For example, let's simplify the following expression  $a = \sqrt{b^2}$

$$\text{simplify}(\sqrt{b^2})$$

$$\text{csgn}(b) \quad b \quad (3)$$

The `csgn()` function returns the sign of its argument. Suppose we want to take the positive solution, i.e. we know that  $b \geq 0$ . We could use the "assuming" command in combination with `simplify`:

$$\text{simplify}(\sqrt{b^2}) \text{ assuming } b > 0 \quad b \quad (4)$$

We can combine assumptions as well like this:

$$\text{simplify}(\sqrt{a^2 \cdot b^2}) \text{ assuming } b > 0 \text{ and } a > 0 \quad a \ b \quad (5)$$

## Solving a system of equations

Consider the following system of equations:

$$\begin{aligned} y + 2x + 2 &= 0 \\ 3y - 4x &= 0 \end{aligned}$$

We'd like to solve this system of equations for  $x$  and  $y$ .

We'll start by defining each equation and assigning it to a label (using "`:=`").

`restart`:

$$\begin{aligned} \text{eq1} &:= y + 2x + 2 = 0 \\ \text{eq1} &:= y + 2x + 2 = 0 \end{aligned} \quad (6)$$

$$\text{eq2} := 3y - 4x = 0$$

$$\text{eq2} := 3y - 4x = 0 \quad (7)$$

This may look a little funny at first. It seems this "equation" has two equals signs. But, the first "`:=`" is only an assignment operator. The label "eq1" is not part of the equation. It just labels the equation. We can now solve our system of equations like this:

$$\begin{aligned} \text{sol} &:= \text{solve}(\{\text{eq1}, \text{eq2}\}, \{x, y\}) \\ \text{sol} &:= \left\{ x = -\frac{3}{5}, y = -\frac{4}{5} \right\} \end{aligned} \quad (8)$$

The first solution (for  $x$ ) is

$$\begin{aligned} \text{sol}[1] & \\ x &= -\frac{3}{5} \end{aligned} \quad (9)$$

Notice that this result is just an equation. It does not store the value  $-3/5$  into a variable  $x$ . To extract the actual value and store it in a variable  $x$ , we can use the right hand side operator `rhs()` like this

$x := \text{rhs}(\text{sol}[1])$

$$\textcolor{blue}{x} := -\frac{3}{5} \quad (10)$$

$y := \text{rhs}(\text{sol}[2])$

$$\textcolor{blue}{y} := -\frac{4}{5} \quad (11)$$

We can now use the variables  $x$  and  $y$  in other calculations:

$x$

$$-\frac{3}{5} \quad (12)$$

$y$

$$-\frac{4}{5} \quad (13)$$

Note: the command **solve()** attempts an analytic solution. We will discuss the command **fsolve()** in a future tutorial, which computes numerical solutions.