Maple Tutorial 4: Differentiation

If you are not familiar with the difference between functions and expressions in Maple, I recommend reading over the Functions, Expressions and Plotting document before continuing.

Don't forget the restart command to clear the variables for the tutorial.

restart **:**

1. Derivatives

Let's suppose we want to calculate the derivative of position with respect to time $\frac{d}{dt}x(t)$. The Maple command to calculate the derivative depends on whether $x(t)$ is an expression or a function.

1.1 Derivative of an Expression

Let's differentiate $\sin(t)$ with respect to t. The simplest method is to use the *diff*( ) command, where our expression to differentiate goes first and the variable we want to differentiate with respect to goes second. The result is an expression we name v for "velocity".

$v := diff(\sin(t), t)$

$$\cos(t) \quad \textbf{(1)}$$

We can evaluate the derivative at a particular time t using the subs() and simplify() commands:

$simplify\left(subs\left(t = \frac{\mathrm{Pi}}{4}, v\right)\right)$

$$\frac{1}{2}\sqrt{2} \quad \textbf{(2)}$$

We could also have defined a variable to hold our expression. The following produces the same result as above. Notice the colon after the sin(t) suppressed output.

$x := \sin(t)$:
$v := diff(x, t)$

$$\cos(t) \quad \textbf{(3)}$$

Higher-order derivatives may be written in two equivalent ways. In the following we calculate the jerk of the particle which is defined as the third derivative of position with respect to time $j = \frac{d^3x}{dt^3}$

$jerk := diff(x, t, t, t)$

$$-\cos(t) \quad \textbf{(4)}$$

or

$jerk := diff(x, t\$3)$

$$-\cos(t) \quad \textbf{(5)}$$

Let's clear the variables before we start the next section:

restart :

1.2 Derivative of a Function

Let's define a function $X(t) = \sin(t)$. In this example we will use lower-case letters for expressions and upper-case letters for functions

$X := t \rightarrow \sin(t)$

$$t \rightarrow \sin(t) \quad \textbf{(6)}$$

Because the *diff*( ) command only works on functions, we need to first convert our function X into an expression by evaluating it at time t like this: $X(t)$. Now we can use the *diff*( ) command on our expression to calculate the velocity:

$v := diff(X(t), t)$

$$\cos(t) \quad \textbf{(7)}$$

If we want to convert our expression v into a function $V(t)$, we use the *unapply*( ) command

$V := unapply(v, t)$

$$t \rightarrow \cos(t) \quad \textbf{(8)}$$

Now, we can evaluate the velocity at a time, say $t = \frac{\mathrm{Pi}}{3}$.

$V\left(\frac{\mathrm{Pi}}{3}\right)$

$$\frac{1}{2} \quad \textbf{(9)}$$

These commands can be combined to directly create a velocity function:

$V := unapply(diff(X(t), t), t)$

$$t \rightarrow \cos(t) \quad \textbf{(10)}$$

Maple also has a differentiation command D( ) that works directly on functions. The first argument is the function we want to differentiate and the second is the independent variable.

$V2 := t \mapsto \mathrm{D}(X)(t)$

$$t \mapsto \mathrm{D}(X)(t) \tag{11}$$

The above result is a function that we can evaluate as usual

$V2\left(\frac{\mathrm{Pi}}{4}\right)$

$$\frac{1}{2}\sqrt{2} \tag{12}$$

To convert our velocity function into an expression, we evaluate V2 with time t plugged in

$V2(t)$

$$\cos(t) \tag{13}$$

Higher derivatives may be specified with a superscript. For example the jerk of $X(t)$ is

$jerk := \mathrm{D}^{(3)}(X)(t)$

$$-\cos(t) \tag{14}$$