3. Analytic Solution of Ordinary Differential Equations

Don't forget the restart command to clear the variables for the tutorial.

restart :

1. Analytically Solve a 2nd Order ODE

To solve an ODE, we following a few steps

Step 1. Define the ODE

Let's solve the simple harmonic oscillator equation $\frac{d^2}{dt^2}x(t) = -\omega^2 x$. We create the derivatives as expressions using the ***diff***( ) command and write the ODE as follows (the double t's mean second derivative). We give the equation the name ode.

$ode := diff(x(t), t, t) = -\omega^2 \cdot x(t)$

$$\frac{d^2}{dt^2} x(t) = -\omega^2 x(t) \quad (1)$$

x(t) will be a function, so we need to specify $x(t)$ in the ***diff***( ) command and not just x.

Step 2. Solve the ODE

Now that we have defined the ODE, we solve it using the ***dsolve***( ) command. The first argument is the name of the equation you want to solve (ode in this case) and the second arguement is the variable you are solving for. Make sure to give the solution a name (I chose "sol").

$sol := dsolve(ode, x(t));$

$$x(t) = \_C1 \sin(\omega t) + \_C2 \cos(\omega t) \quad (2)$$

Because we didn't specify the initial conditions, Maple includes the constants of integration ***_C1*** and ***_C2***.

To remove the integration constants, we can specify the initial conditions. For a second-order ODE, this means defining the initial position x_0 and the initial velocity v_0. Notice that we use the D() command to take the time derivative of position since x is a function.

$ic1 := x(0) = x0$

$$x(0) = x0 \quad (3)$$

$ic2 := D(x)(0) = v0$

$$D(x)(0) = v0 \quad (4)$$

We can modify the ***dsolve***( ) command to include the initial conditions. The name of the ODE is grouped together with the initial conditions using square brackets.

$sol := dsolve([ode, ic1, ic2], x(t))$

$$x(t) = \frac{v0 \sin(\omega t)}{\omega} + x0 \cos(\omega t) \qquad (5)$$

Now our solution is in terms of the initial position ***x0***. The result is an equation in the form ***x(t) = (an expression)***. If you want to work with the expression on the right hand side (rhs) of the equals sign use the command $xx := rhs(sol)$.

To find the velocity as a function of time, we differentiate the solution using the ***diff*** ( ) command

$diff(sol, t)$

$$\frac{d}{dt} x(t) = -\sin(\omega t)\, \omega\, x0 + \cos(\omega t)\, v0 \qquad (6)$$

Step 3. Graph the solution

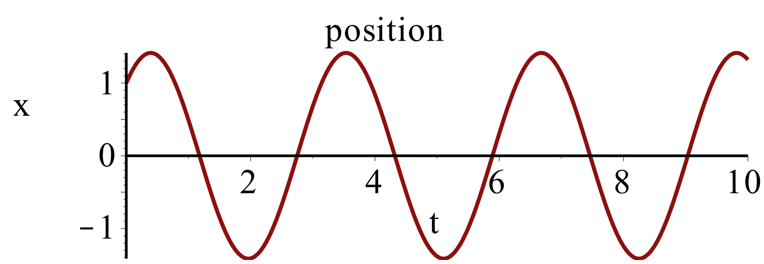
the right-hand-side of the solution ***rhs(sol)*** is an expression, we can use the ***subs*** ( ) command to insert the values

$SOL := subs(\omega = 2, x0 = 1, v0 = 2, rhs(sol))$

$$\sin(2\,t) + \cos(2\,t) \qquad (7)$$

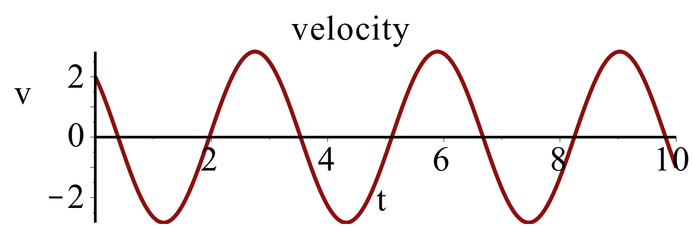
We now have the solution in a format that we can plot

$plot(SOL, t = 0..10, labels = [\text{"t"}, \text{"x"}], title = \text{"position"})$



To plot the velocity as a function of time try this:

$plot(diff(SOL, t), t = 0..10, labels = [\text{"t"}, \text{"v"}], title = \text{"velocity"})$



Just the Code

Here are the equations without the narrative. We use colons at the ends of most statements to suppress output.

$restart:$

$ode := diff(x(t), t, t) = -\omega^2 \cdot x(t):$

$ic1 := x(0) = x0:$

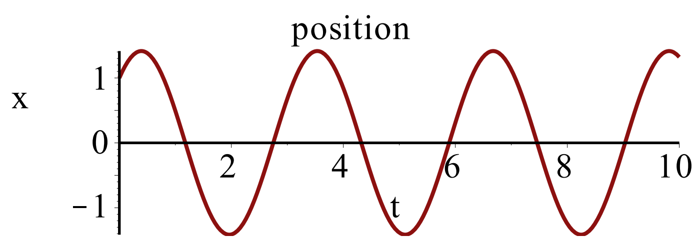
$ic2 := D(x)(0) = v0:$

$sol := dsolve([ode, ic1, ic2], x(t)):$

$SOL := subs(\omega = 2, x0 = 1, v0 = 2, rhs(sol))$

$$\sin(2\,t) + \cos(2\,t) \quad \textbf{(8)}$$

$plot(SOL, t = 0..10, labels = [\text{"t"}, \text{"x"}], title = \text{"position"})$



$plot(diff(SOL, t), t = 0..10, labels = [\text{"t"}, \text{"v"}], title = \text{"velocity"})$